Towards a formal regimentation of the Navya-Nyāya technical language I

Jonardon Ganeri*

Department of Philosophy, University of Liverpool, 7 Abercromby Square, Liverpool L69 7WY, United Kingdom

E-mail: jonardon@liverpool.ac.uk

Navya-Nyāya is an early modern Indian system of philosophical analysis. It was founded by Udayana (c. 1050 CE), developed by Gaṅgeśa (c. 1200 CE), and reached its peak in the works of authors including Raghunātha (c. 1500 CE), Jagadīśa (c. 1600 CE) and Gadādhara (c. 1650 CE).[1] The school is notable for its development of a technical language, by means of which it clarified many philosophical questions in the traditional Indian debate. This technical language rapidly became the standard idiom for academic works in Sanskrit, not only in philosophy, but in grammar, poetics, law, and other branches of study as well. A careful analysis of the conceptual framework and expressive power of the Navya-Nyāya technical language is therefore of considerable importance in the modern study of the Indian academic literature.

1.1

Ordinary Sanskrit is regarded by Navya-Nyāya as an imperfect vehicle for philosophical discourse, mainly because it is infested with ambiguity. It is not just the presence of ambiguities in the lexicon, homonymous words like *saindhava* (which can mean either salt or a horse), for it is always possible to eliminate such terms in favour of words which are not ambiguous. The greater problem derives from the absence in ordinary Sanskrit of any systematic or compulsory use either of articles or quantifier expressions. In English, the combination of an "applicative" [10, p. 73], that is, an expression like "a", "the", "all", "some", or "most", with a substantival general term such as "pot" or "cow" forms a descriptive or quantified phrase. In Sanskrit, however, an inflected noun or noun phrase occurring by itself often has the same syntactic role. So, for example, the sentence *ghaṭo nīlaḥ* "pot [is] blue" might mean either "the pot is blue", "some pot is blue" or "every pot is blue". Similarly, a phrase like "cause of fire" might signify the cause of a certain fire ("The cause of [the] fire is unknown"), the cause of a fire ("carelessness causes fire"), or the cause of all fire ("a cause of every fire is heat").

*I should like to express my gratitude to the two anonymous referees, whose comments and advice have been invaluable. I should also like to thank Benedikt Löwe.

[1] For the history, cf. [18, 1].

Mihir K. **Chakraborty**, Benedikt **Löwe**, Madhabendra Nath **Mitra**, Sundar **Sarukkai** (*eds.*). Logic, Navya-Nyāya & Applications. Homage to Bimal Krishna Matilal. College Publications, London, 2008. Studies in Logic 15. pp. 109–124.

Received by the editors: 1 May 2007; 28 March 2008, 16 May 2008.
Accepted for publication: 11 November 2008.

1.2

In some contexts then, an inflected noun has the role of a definite description, as for example in "[The] doctor is coming". A classical dispute in Indian philosophy of language is whether the semantic value of "(the) doctor" in a sentence like this is an individual or a universal. These two positions are called Meaning Particularism (*vyaktiśaktivāda*) and Meaning Universalism (*jātiśaktivāda*) respectively. The former view represents a doctrine that nouns have a genuine referring use, and it is within discussions of this view that the theory of reference developed in India. The Universalist, on the other hand, whose approach is in some ways nearer to the standard Russellian treatment of definite descriptions, explains the referential use of nouns by appeal to pragmatic constraints (*lakṣaṇā*) on the interpretation of sentences (for further discussion, cf. [8]).

1.3

In other sentences, a noun can have an existential or universal force; for example "Bring [a] pen" or "[A] cow should not be kicked". Faced with the problem of accounting for this use of nouns, those Indian semanticists who endorsed Particularism argued that a noun has a systematic ambiguity in semantic role, sometimes taking an individual as its value, and sometimes a universal or class. This approach was shared by the early grammarians such as Patañjali as well as the early Naiyāyikas. I shall try to show how this idea comes to be formalised in the Navya-Nyāya technical language.

1.4

When two or more nouns occur in the same sentence, there is a possibility of 'scope' ambiguities. Suppose one forms a phrase "cause of smoke". This might mean cause of a particular body of smoke, cause of a some or other smoke, or else cause of all smoke. Now, when such a phrase is combined with another noun to form a sentence, e.g., "Fire is cause of smoke", the three-fold ambiguity of each noun leads to nine different readings of the sentence. In two of these readings "All fire is (the) cause of some smoke" and "Some fire is (the) cause of all smoke", there is a further ambiguity. The first, for example, might mean "Each fire is (the) cause of some smoke or other" or "There is a body of smoke which is the effect of every fire". This scope ambiguity is not the result of any putative ambiguity in the nouns, but is due to an ambiguity in the syntax of the language.

1.5

When sentences contain relational expressions, there is room for still another sort of ambiguity. Compare the sentences "Pothood is in a pot" and "The cat is in the kitchen". In the first case, the relational expression "in" (or the locative post-noun) indicates the relation of inherence, but in the

second it indicates the relation of containment. The English expression "is" is similarly ambiguous, sometimes indicating identity, sometimes existence, and sometimes predication. This kind of ambiguity concerns the semantic role of the relational expression in the sentence [20, p. 136].[2] Naiyāyikas claim that the negation particle *na* "no" suffers from ambiguity in this way: it sometimes indicates a "mutual absence" (*anyonyābhāva*), i.e., the negation of an identity, but sometimes a "relational absence" (*saṃsargābhāva*), i.e., the negation of a predication (cf. [15, p. 94–95]; [14, pp. 54–55]).

2.1

If an ordinary language "with all its ambiguities and abominable syntax" (Russell, quoted in [20, p. 134]) is ill-suited for the careful formulation of philosophical doctrine, the alternative is to construct a formal artificial language free from these defects. This is exactly what Navya-Nyāya does. Authors such as Frege, Russell, and Quine, all of whom introduce artificial or formal languages, differ in their opinions as to the relation between the formal language and ordinary languages, and it is not entirely clear how Navya-Nyāya conceives of the relation between ordinary Sanskrit and its technical language.[3] Minimally, we should suppose it to be such that corresponding to each of the various possible readings of an ordinary Sanskrit sentence, there should be just one sentence in the formal language.

2.2

The Nyāya language includes a small number of logical words, especially "substratum" or "locus" (*adhikaraṇa*, *ādheya*) and its inverse "occurrence" (*vṛtti*), "conditioner" (*nirūpaka*), "delimitor" (*avacchedaka*) and "absentee" or "counterpositive" (*pratiyogin*), together with a non-logical vocabulary of terms and relation-expressions.[4] Accompanying the language, there is a formalised ontology, which is a modified version of the Vaiśeṣika system of categories. Modern interpreters of Nyāya disagree on whether the Nyāya ontology is extensional or intensional (cf. [21, 14, 5, 11, 15]). On the one hand, Nyāya exploits the various abstraction devices in Sanskrit, and speaks of pothood or cause-of-fire-hood where we might speak of the class of pots

[2] Or, if we agree with the Nyaya that the relational element is indicated, not by an explicit expression, but by grammatico-syntactic features (*ākāṅkṣā*) of the sentence, we must locate the ambiguity there.

[3] [22, pp. 5–6] usefully distinguishes between three historical positions, according to which the formal language is regarded as an *extension*, an *improvement*, or a *reform*, of natural language. The Navya-Nyāya attitude seems to me to have most in common with the view expressed by the later Wittgenstein [24, § 19], that the relationship between a formal language and natural language is akin to the relationship between a new suburb, "with regular streets and uniform houses" and the ancient city, "a maze of little streets and squares." Cf. also [4, pp. 30-33], [12, pp. 9–15], and [2].

[4] Cf. [14, pp. 28–85], [15, pp. 3–98], [6], [21, pp. 16–35], [12], and [23, pp. 24–35].

or the causes of fire. Yet, when one examines the role of such abstract properties in the theory, it is very often only their extension which is semantically relevant. When developing the Nyāya system, I shall follow the strategy of using a set-theoretic ontology as far as possible. It seems that Nyāya tries in the main to avoid disputes about ontology, and develops a theoretical language which can be used even by those who do not share its ontological dispositions (cf. [16, p. 66], [3, p. 201]). The mention of properties in the Nyāya formal language seems then to be mainly pleonastic. In particular, the abstraction device "-*tva*" is used freely, not restricted to the naming of genuine properties alone: a Naiyāyika happily reparses the sentence "the pot has a long neck and a conch-shaped handle" as "long-neck-conch-shaped-handle-ness is located in the pot". Its commitment to a principle of ontological parsimony prevents Navya-Nyāya from agreeing that every such operation generates the name of a genuine property.[5]

2.3

The Nyāya language is not a symbolic one. It does not, for example, employ variables, although "dummy singular terms" [15, p. 23] like *ghaṭa* ("pot"), as well as the pronouns *tat* "that" and *sva* "own-" sometimes function in the same way. The sentences in the Nyāya language therefore resemble "long-hand" versions of symbolic formulae and as a result are notoriously cumbersome. In principle, however, there is no reason why the Nyāya language cannot be symbolised, and I shall attempt to construct a symbolic notation for a fragment of the language here. We can denote this fragment NN. As long as we restrict our attention to the sentences for which a set-theoretic ontology is sufficient, this fragment should be equivalent to some part of first-order predicate logic, and it will be interesting to find out which part this is.

3.1

I shall now give an informal presentation of the fragment of the Navya-Nyāya language NN, reserving a more formal treatment for [9]. Informally, we can say that syntax for the fragment NN of the Nyāya technical language is built up from the following components:

(1) There is a set of **primitive terms**, such as *ghaṭa* ("pot"), *go* ("cow"),

[5] In [13], Guhe argues that the Navya-Nyāya attitude towards the ontology of properties is similar to that of George Bealer's property theory. Their commitment to a principle of ontological parsimony (which they name *lāghava*; cf. [15, p. 83]) is the reason I doubt if the liberal ontology of Bealer's property theory is necessary for interpreting Navya-Nyāya. It is, however, possible, that a sparser intensional system will be what is needed. My ambition in this paper is to see how much of Navya-Nyāya can be understood on a purely extensionalist basis; we can then ask what is the leanest intensional addition necessary to handle the residue.

etc. i.e., the nouns or uninflected nominal stems (*nāman*, *prātipadika*). We shall use the Roman letters "*a*", "*b*", etc. for these primitive terms.

(2) There is an **abstraction functor** *-tva* or *-tā* ("-ness", "-hood"), the operation of which on a primitive term like *ghadta* "pot" gives rise to an abstract term *ghaṭatva* "pothood". I shall use the Greek letters "α", "β", etc. for abstract terms.

Some Naiyāyikas employ a second abstraction functor — *vyaktitva* ("-individual-hood"), e.g., *ghaṭavyaktitva* "pot-individual-hood". This functor allows us to replace any particular use of a primitive term with a corresponding abstract term. Nyāya sometimes uses this device to eliminate primitive terms from its technical language. It partially resembles Quine's elimination of proper names like "Socrates" in favour of predicates like "*x* Socratises" [19, p. 181], or rather, "to Socratise".

(3) There is a set of **relational abstract expressions**, such as "locushood", "causehood", "cousinhood", "pervadedness", some logical and some non-logical. There is also a corresponding set of inverse relational abstract expressions, "superstratumhood", "effecthood", etc. I shall use bold letters, e.g., "**R**" for these expressions.

(4) There is a **conditioning operator**, which combines a relational abstract expression with a term (of any kind) to form a term, such as "locushood-conditioned-by-pot" (*ghaṭa-nirūpitādhraṇatā*) or "causehood-conditioned-by-smokehood" (*dhūmatva-nirū-kāraṇatā*). I shall call such expressions "relational terms", because they are terms derived from a relation by specifying its adjunct. Nyaya often abbreviates them to "locushood-to-pot" (*ghaṭīyādhāratā*) etc. (cf. [14, p. 83]). A conditioned relational abstract is represented here by writing the conditioner letter on the right hand side of the relational abstract expression, thus "**R**β".

(5) There are two kinds of **sentence-forming operator**. One combines a relational term "**R**β" with another term to form a sentence "a**.R**β" or "α**.R**β". This operator is named "location" or "residence" (*niṣṭhana*) if the term is primitive, and "delimitation" (*avacchedana*) if it is abstract. For example "locushood-conditioned-by-pot is resident in ground" (*ghaṭa-nirūpitādhāraṇatā sā bhūtalaniṣṭhā*, or *ghaṭa-nirūpitādhāratāśrayaṃ bhūtalam*) or "causehood-conditioned-by-smokehood is delimited by firehood" (*dhūmatva-nirūpita-kāraṇatā sā 'gnitvāvacchinnā*).

The second sentence-forming operator, colocation (*samanadhikaranya*), represented here by a colon, combines an abstract term "α" with a relational term "**R**β" to form a sentence "α**:R**β".

(6) There is a **negation functor** "-absence" (*atyantābhāva*), which forms negative terms such as "pot-absence" (*ghaṭābhāva*) from terms. By definition, the negative term "pot-absence" is identical with the relational abstract term "absenthood-conditioned-by-pot" (*ghaṭa-nirūpitānuyogitā*), where "absenthood" (*anuyogitā*) is a logical relational abstract expression. We shall write "absenthood" as "**N**", and negative terms as "*a*-absence" or "**N***a*" etc. There is also a **sentence negation** "not" (*na*). Thus, "causehood-conditioned-by-smokehood is not delimited by firehood" (*dhūmatva-nirūpita-kāraṇatā sā 'gnitvānavacchinnā*). Nyāya avoids sentential negation wherever it can (cf. [17, p. 116]), but cannot eliminate it altogether. I shall isolate the point at which its introduction is necessary below.

The syntax of NN thus consists of relational abstract expressions, various different kinds of term expressions —primitive, relational, abstract, and negative— and a negation particle.

4.1

For the semantics of NN, I shall, as already stated, only draw upon a set-theoretic ontology. Nyāya does not use set-theoretic notions like set inclusion or set membership, but prefers to talk instead of properties occurring in objects, or co-occurring with other properties etc. It is for this reason that it is sometimes said to have a "property-location language". However, since its semantic vocabulary is often clearly extensional (properties which are equipollent (*samaniyata*) are in many cases identified), there is some motivation to using a set-theoretic notation[6]. We can then assign, to each expression in the syntax, an element or set as follows.

(1-2) As noted above, the Nyāya regard nouns sometimes as functioning like singular referring expressions. When thus used, they will share with indexicals the property of taking a different referent depending on the context of use. To each occurrence of a primitive term like "pot" is therefore assigned an object P, such that P belongs to the set p of pots. And to the corresponding abstract term "pothood" is assigned the property pothood. In keeping with our simplifying restriction, let us assign to such an expression the set p of pots[7]. It is now possible for any particular use of an noun-phrase in the ordinary language to be mapped either to a primitive term having a particular value, or else to the corresponding abstract term, whose value is a class.

[6]Note, however, Ingall's reservation [14, p. 50], and [3, p. 290], as well as my comments in Footnote 2 above.

[7]Recall Kātyāyana's aphorism under Pāṇini-sūtra 5.1.119: "the abstraction suffixes [i.e., -hood, -ness, -ity] such as -*tā* and -*tva* (added to nominal stems) 'express' (*abhi* $+\sqrt{}$*dhā*) only those qualities (*guṇa*) on the basis of which the nominal stems are used to refer to things".

Corresponding to each occurrence of a primitive term, there is an abstract term formed by the application of the individuality abstraction functor, for example, the term "pot-individual-hood" (*ghaṭavyaktitva*; cf. [15, p. 57]). If the value assigned to a particular occurrence of the primitive term is P, then the set assigned to this abstract term is the unit set $\{P\}$, i.e., the property of being this very pot.

(3-4) Let us next introduce a number of relations. In set theory a relation is a subset of the Cartesian product of two sets, A and B, the former being the range and the latter the domain of the relation. In other words, a relation is a set of ordered pairs. Naturally, there is a degree of anachronism in using such a notion of relation to explicate the Nyāya system. Yet since the Nyāya claim that a relation is made up of a collection of relation-particulars, each of which is individuated by specifying the two relata (cf. [15, pp. 33–34]), the anachronism may be justifiable. Now given any relation, we can form a series of sets, the extensions of relational properties, as follows. Suppose that an object b is in the domain B of the relation R. Then we can form the set of elements in A which are related by R to b. Similarly, given a set $\beta \in B$, we can form the set of elements in A which are related by R to some element in β. In the standard terminology of relations, this set will be the image of β under the inverse of R. Now, if b is an object assigned to an occurrence of the primitive term "b", or β is the set assigned to an abstract term "b-hood", then we shall say that the set thus formed is the set assigned to the relational abstract term "R-hood-conditioned-by-b(-hood)". For example, the set assigned to the relational abstract term "causehood-conditioned-by-pot-hood" (*ghaṭatva-nirūpita-kāraṇataṭa*) comprises those objects which are the cause of a pot. Let such sets be assigned to "$\mathbf{R}b$" or "$\mathbf{R}\beta$".

Given any relation R, it is possible to form an inverse relation R^{-1} such that $yR^{-1}x$ iff xRy.

(5) If "a" is a token of a primitive term, and "$\mathbf{R}\beta$" is a relational abstract term, then the sentence "$\mathbf{R}\beta$ is resident in a" (or equivalently "a is the locus of $\mathbf{R}\beta$"), i.e., "$a.\mathbf{R}\beta$", is true iff the object assigned to "a" is a member of the set assigned to "$\mathbf{R}\beta$". Similarly, if "a-hood" is an abstract term, then the sentence "$\mathbf{R}\beta$ is *delimited by* a-hood", i.e., "$\alpha.\mathbf{R}\beta$", is true iff the set assigned to "a-hood" is contained in the set assigned to "$\mathbf{R}\beta$"[8]. For example, the sentence "causehood-conditioned-by-smokehood is delimited by firehood" is true iff the set of fires is a subset of the set of causes of smoke.

[8]This is the definition of a limitor as "that which occurs in no more [than the abstract]" (*anatirikta-vṛtti*). Cf. [15, p. 76].

The sentence "a-hood is co-located with $\mathbf{R}\beta$" is true iff the intersection of a-hood with $R\beta$ is non-empty, e.g., iff there is a fire which is the cause of smoke.

(6) The Nyāya treatment of negative terms is a little peculiar. Nyāya in fact expands its ontology to include, for every object such as a pot P, an "absentee" or "anti-object" (*abhāva*), an absence of the pot P for example. It expresses the fact that P is not on the table by saying that the anti-object absence-of-P is on the table. Given a token primitive term such as "pot", we can form a negative term "pot-absence" by means of the relational abstract term "absenthood-conditioned-by-pot": the set assigned to "pot-absence" is the set of absences which are absences of the pot P. In fact, since the absence relation is assumed to be one-one, there is only one such absence, and it is called a "specific" absence (*viśeṣābhāva*).[9] Given an abstract term such as "pothood", we can form the negative term "absenthood-conditioned-by-pothood" (unfortunately also written as "pot-absence"), to which is assigned the set of absences which are absences of some pot or other. Nyāya also says that there are "generic absences" (*sāmānyābhāva*), the absence of any pot, for example. We shall see later how such generic readings of "absence of pot" are obtained in the Nyāya language.

4.2

This completes the semantics of NN, but I should like to note a frequently encountered extension. When the conditioner of the relational abstract term "R-hood" is a primitive term "b", Nyāya sometimes reformulates "R-hood-conditioned-by-b" as "R-hood-conditioned-by-R^{-1}-hood-resident-in-b" (e.g., *ghaṭa-niṣṭha-kāryatā-nirūpita-kāraṇatā*). Similarly, when the conditioner is an abstract term "b-hood", Nyāya sometimes reformulates "R-hood-conditioned-by-b-hood" as "R-hood-conditioned-by-R^{-1}-hood-delimited-by-b-hood" (e.g., *ghaṭatvāvacchinna-kāryatā-nirūpita-kāraṇatā*)[10]. The use of the terms "conditioned by", "resident in" and "delimited by" in these neologisms are distinct from, although related to, their use above as term- and sentence-forming operators. The point to these reformulations is as follows. Suppose that an object a is in the domain A of the relation R. Then we can form the set of elements in B which are related by R to a. Similarly, given a set $\alpha \in A$, we can form the set of elements in A which are related by R to an element in α. Now, if a is an object assigned to a token primitive term "a", or a is the set assigned to an abstract term "a-hood", then we shall say that the set thus formed is the set assigned to the relational term "R-hood-resident-in-a" or "R-hood-delimited-by-a-hood", respectively. For example, the set assigned to the relational abstract term

[9] Cf. [14, p. 56].
[10] Cf. [14, p. 46] and [15, p. 80].

"causehood-delimited-by-fire-hood" (*agnitvāvacchinna-kāraṇatā*) comprises those objects which are the effect of a fire. Let such sets be assigned to the terms "$a\mathbf{R}$" or "$\alpha\mathbf{R}$", respectively. The reformulation relies on the identity of $\mathbf{R}b$ with $b\mathbf{R}^{-1}$, and of $\mathbf{R}\beta$ with $\beta\mathbf{R}^{-1}$. Nyāya sometimes calls the notion of delimitation in such a reformulation delimitation-by-conditioning (*nirūpitatva-saṃbandhāvacchinna*), to distinguish it from the sentence forming delimitation-by-residence (*niṣṭhatva-saṃbandhạvacchinna*) introduced earlier[11]. Conditioning delimitors, by contrast, are 'fragments' of a term. The reformulation leads to a pleasant simplification in certain cases. When the relation R is one-one, or when the relation restricted to the sets α and β is itself a sub-relation, then $\alpha.\mathbf{R}\beta$ iff $\beta.\mathbf{R}^{-1}\alpha$. In such a case, we can say that $\mathbf{R}$ as conditioned by $\beta\mathbf{R}^{-1}$ is delimited by α iff $\mathbf{R}$ as conditioned by $\alpha\mathbf{R}^{-1}$ is delimited by β (e.g., *ghaṭatvāvacchinna-kāryatā-nirūpita-kāraṇatā sā daṇḍatvāvacchinnā*, and vice versa). However, I shall ignore this extension in what follows.

5.1

We are now ready to see how an ordinary Sanskrit sentence, e.g., "pot is on table" (or the Nyāya example "*bhūtale ghaṭaḥ*"), is disambiguated in the Nyāya technical language. Nyāya resolves the ambiguity of semantic role in the relation expression "on" by saying that contacthood, rather than inherencehood etc. is the "limiting relation" (*avacchedaka-saṃbandha*) of the sentence[12]. Having done that, it needs to distinguish the eleven distinct readings isolated in § 1.4, which it does as follows.

(1) "A particular table t is the locus of a particular pot p". This would be expressed by saying that the relational abstract locushood conditioned by the pot p is resident in the table t. In our symbolic notation, we should write "$t.\mathbf{L}p$", where "t" represents the token primitive term "table" whose value is t, "p" the term "pot" whose value is p, and "$\mathbf{L}$" the relational abstract locushood. "$t.\mathbf{L}p$" is true iff $t \in \mathbf{L}p$, the class of objects on which p is located.

(2) "t is the locus of some pot". The Nyāya paraphrase is: the relational abstract locushood conditioned by pothood is resident in t. Symbolically, "$t.\mathbf{L}\pi$", where "π" represents the class of pots. "$t.\mathbf{L}p$" is true iff $t \in \mathbf{L}\pi$, the class of objects on which some pot is located. A sentence for which (2) might be the most natural construal is "[The] mountain possesses fire" (*parvato vahnimān*).

(3) "t is the locus of all pots". This is turned around to read "Every pot occurs on t", and is paraphrased as: occurrenthood-to-t is limited by pothood, where occurrenthood (*vṛtti*, *ādheyatā*) is the inverse of locushood.

[11] Cf. [14, p. 50] and [15, p. 75].
[12] Cf. [14, p. 51] and [15, p. 77].

In our notation, this reads as "$\pi.\mathbf{L}^{-1}t$", which is true iff $\pi \subseteq \mathbf{L}^{-1}t$, i.e., the class of pots is a subset of the class of things located on t. Nyāya sometimes says that t is the "generic locus" of pot.[13] Reading (3) is especially natural when absence is involved. As Ingalls notes,

> an absence the [absenteehood (*pratiyogitā*, i.e., the inverse of absenthood)] to which is limited by a generic character or by a property common to several entities is termed a generic absence (*sāmānyābhāva*). Notice that generic absences have the effect of negating all particulars of a given class. [14, p. 56]

Clearly, we should not think of generic loci or generic absences as a special kind of entity.

(4) "Some table is the locus of p" (i.e., "p is on a table"). Nyāya says here that occurrenthood-to-tablehood is resident in p (p possesses the property of occurring on a table.). Symbolically, "$p.\mathbf{L}^{-1}\tau$" ("τ" is the abstract term "tablehood", whose extension is the set τ of tables), which is true iff $p \in \mathbf{L}^{-1}\tau$, the set of objects which occur on a table.

(5) "Every table is the locus of p". Locushood-to-p is limited by tablehood, i.e., "$\tau.\mathbf{L}p$", which is true iff $\tau \subseteq \mathbf{L}p$. We find in the early Naiyāyika Vātsyāyana's discussion of semantics, the sentence "[A] cow should not be kicked [by you]", which may very well serve as an example of this reading.

(6) "Some table is the locus of some pot". Nyāya would say here that locushood to pothood is co-located with tablehood. I.e., "$\tau : \mathbf{L}\pi$", which is true iff $\tau \cap \mathbf{L} \neq \varnothing$.

(7) "Every table is the locus of some pot". This has two readings: (7i) "every table has some pot or other on it", and (7ii) "there is a pot which is on every table". Nyāya expresses the first reading by saying that locushood to pothood is delimited by tablehood, i.e., "$\tau.\mathbf{L}\pi$", which is true iff $\tau \subseteq \mathbf{L}\pi$, (the set of tables is a subset of the set of things with pots on). We might borrow another of Vātsāyana's sentences, "[A] cow is born of [a] cow" to illustrate this reading. It is also a reading closely connected with the Naiyāyikas' notion of pervasion (*vyāpti*). For example, "fire pervades smoke" means that every locus of smoke is also a locus of fire. I shall discuss (7ii) below.

(8) "Some table is the locus of every pot". Again, there are two readings, (8i) "every pot is on some table or another", and (8ii) "there is a table which is the locus of every pot". The first reading is naturally expressed by saying that occurrenthood to tablehood is bound by pothood, i.e., "$\pi.\mathbf{L}^{-1}\tau$". (8ii) requires a similar treatment to (7ii).

[13] Cf. [14, p. 50].

(9) "Every table is the locus of every pot". This too will be discussed below.

6.1

The structure of the Nyāya formal language might be further clarified if we can set up a "translation manual" between NN and some fragment of the predicate logic, presumably a fragment containing diadic predicates and quantifiers. The examples discussed above suggest the form such a translation manual might take. From readings such as (1), it is clear that each occurrence of a primitive term will translate into an individual constant. Consider now a sentence like "fire causes smoke" (i.e., reading (7i) above). The Nyāya form first the expression "causehood-conditioned-by-smokehood", which translates into the open sentence "$(\exists x : \text{smoke})(y \text{ causes } x)$", where "$: f$" indicates a restriction on the domain of quantification to things that are f. The original sentence is then paraphrased as "causehood-conditioned-by-smoke is delimited by fire", which translates as

$$\text{“}(\forall y : \text{fire})(\exists x : \text{smoke})(y \text{ causes } x)\text{”}.$$

So a conditioner maps to an existential quantifier, whose domain is restricted to the class assigned to the conditioner, and which binds the second place of a diadic predicate. Similarly, a delimitor maps to a universal quantifier, whose domain is restricted to the class assigned to it, and which binds the first place of a diadic predicate. It is clear from the way sentences are constructed in NN that the universal quantifier corresponding to the limitor always has wider scope than the existential quantifier corresponding to the conditioner. Finally, the co-location operator will translate into an existential operator binding the first place of the diadic predicate, for a sentence like (6), "locushood-conditioned-by-pothood is co-located with tablehood" ("$\tau : \mathbf{L}\pi$") translates to "$(\exists x : \tau)(y : \pi)(x \; L \; y)$". In this way the technical language formalises an ambiguity in the semantics of an ordinary noun-phrase, by translating it into either a token primitive term or an abstract term, and assigning to it either an individual or a class[14].

6.2

The ordinary language Sanskrit sentence rendered as "pot is not in the room" has three distinct readings. It might mean that a certain pot p is not in the room; or that there is a pot which is not in the room; or that no pot is in the room. In everyday Sanskrit, the third reading is usually the most naturally intended one. In NN, we can form from a primitive term

[14]The idea that nouns are ambiguous in this way was first clearly stated by the *tadvat* theorists, Uddyotakara and Jayanta. See their comments under Nyāyasūtra 2.2.66, and [16, pp. 67–69].

"pot", a negative term "pot-absence". We can also form a negative term from the abstract term "pothood", also expressed as "pot-absence". These two terms are by definition equivalent to the relational terms "absenthood-conditioned-by-pot" ("$\mathbf{N}p$") and "absenthood-conditioned-by-pothood" ("$\mathbf{N}\pi$"), where the relation of absence is a one-one relation between any entity and its negative entity or absentee. The first reading is now expressed as "$t.\mathbf{L}(\mathbf{N}p)$", i.e., "$t$ is a member of the set of loci of absentees of p", i.e., "$-tLp$". The second reading is expresses as "$t.\mathbf{L}(\mathbf{N}\pi)$", i.e., "$t$ is a member of the set of loci of absences of a pot", i.e., "t is a member of the set of objects which are such that there is a pot for which it is not the locus", i.e., "$(y : \pi)(-tLy)$". Note how this shows that the absence relation, with its corresponding negative terms, is equivalent to a negation which always takes narrowest scope. To catch the third, and most natural, reading of the sentence, i.e., "No pot is on the table" or "The table is the locus of the absence of all pots", Nyāya makes pothood the delimitor of absenteehood (*pratiyogitā*), the inverse of absenthood.[15] The obvious candidate is "$\pi.\mathbf{N}^{-1}(\mathbf{L}^{-1}t)$", i.e., "absenteehood-conditioned-by-superstratumhood-conditioned-by-table is limited by pothood" (*bhūtala-nirūpita-ādheyatā-nirūpita-pratiyogitā sā ghaṭatvāvacchinnā*). This expands as "the set of pots is a subset of absentees for which there is an absence in the set of superstrata of t", i.e., "the set of pots is a subset of the set of objects whose absence is located on t", i.e., "$(x : \pi)(-tLx)$". We might note that, as long as negative terms are only used for the adjunct of another relation, the negative objects are "virtual" entities; they are always quantified out of the final sentence. Matilal exploits this fact to construct a semantics in which every property has a "presence-range" and an "absence-range", corresponding to the set of loci of the property and the set of loci of the absence of the property [17, pp. 112*sqq*].

6.3

The problem of scope ambiguity is usually illustrated by a sentence like "Everybody loves somebody". It is possible to read this sentence in two ways, as saying that given any person, there is someone who loves them, or as saying that there is a person who is loved by everybody. In this second reading, the existential quantifier precedes the universal quantifier. It therefore poses a problem for the Nyāya formal language, in which the universal quantifier or limitor always has widest scope. However, suppose we consider the third 'generic' reading of "pot is not on a table". The NN expression of this is "$\pi.\mathbf{N}^{-1}(\mathbf{L}^{-1}t)$", i.e., "absenteehood-conditioned-by-superstratumhood-conditioned-by-table is limited by pothood", i.e., "the set of pots is a subset of the set of objects whose absence is located on t", i.e.,

[15] Cf. [15, pp. 80–81].

"$(\forall y : \pi)(\exists x : \tau)(-tL^{-1}x)$". If this is not true, then there is a table which is not the locus of the absence of any pot, i.e., a table which is the locus of every pot. So the second reading can be expressed as "not $\pi.\mathbf{N}^{-1}(\mathbf{L}^{-1}\tau)$", i.e., "absenthood-conditioned-by-occurrenthood-conditioned-by-pothood is not delimited by tablehood". In the predicate calculus, this result can be expressed via the theorem: $(\exists y : \beta)(\forall x : \alpha)(xRy) = -(\forall y : \beta)(\exists x : \alpha)(-xRy) = -(\forall y : \beta)(\exists x : \alpha)(-yR^{-1}x)$. So with the help of a negation which always takes narrowest scope (the term negation) and one which always takes widest scope (the sentence negation), we can express the mixed readings. An exactly analogous tactic will obtain (9) from (6). It seems that only in such cases is a sentential negation ineliminable.

7.1

The Nyāya language NN is equivalent to a quantified language (NN*) in which each sentence is constructed as follows. (i) There is a dyadic predicate "$\_\ R\ \_$". (ii) A negation $(-)$ taking narrowest scope optionally occurs next. Thus "$(-)(\_\ R\ \_)$". (iii) The next step is to fill the second place of the predicate, either with a constant or with a variable bound by an existential quantifier, whose range is restricted to a certain set β. We might, for simplicity, use the individualisation device to eliminate the constants in favour of bound variables. This quantifier has wider scope than the negation in (i) but narrower scope than anything else. Thus "$(\exists y : \pi)(-)(\_Ry)$". (iv) Next, the left hand place is bound, either by a constant, or by a restricted universal quantifier or by a restricted existential quantifier. Thus "$(\forall x : \tau)(\exists y : \pi)(-)(xRy)$" or "$(\exists x : \tau)(\exists y : \pi)(-)(xRy)$". (v) The last step is the optional insertion of a negation which takes largest scope. These five steps correspond to forming a relational abstract, conditioning it with a term (possibly negative) to form a relational term, and forming a sentence using delimitation or co-location (possibly negated). It follows that every sentence in this language (NN*) has the structure $(-)(\forall/\exists)(\exists)(-)(\_R\_)$. Unlike predicate calculus, the order of the various components is fixed. However, it seems possible to show that every sentence composed from a diadic predicate, one or two quantifiers, and negation, with no restrictions on the order in which these elements occur, is equivalent to a sentence having the structure of the sentences in NN*. For the formula $(-\forall = \exists-)$ permits any sentence of the form $(\exists\forall)$ or $(\forall\forall)$ to be transposed into one of the form $(\forall\exists)$ or $(\exists\exists)$, respectively, and also permits the transformation of any sentence in which a negation occurs between two quantifiers into one having only narrow or wide scope negation, appropriately inverting the diadic predicate if necessary. Also, a restricted quantifier can be replaced by an unrestricted quantifier together with an appropriate predicate. So the language NN* is equivalent to that fragment of the predicate calculus

whose sentences take the form "$(Fx \wedge Gy \wedge xRy)$", quantified and negated according to taste. Cf. [9] for a more formal treatment.

7.2

The language NN seems to capture some of the logical apparatus used by the Navya-Nyāya authors. I should not claim more than that. The Nyāya authors themselves do not draw such a sharp distinction between terms and sentences as is done in NN, and do not, as far as I am aware, show much interest in the problems of scope ambiguity (and hence understate the need for a sentence negation). There are also many other Nyāya technical notions, for example to do with the concatenation of relations and terms, identity, etc. Moreover, the use of the technical vocabulary varies a little from author to author. And often the language is used in only a semi-formal way, especially when used by non-Nyāya authors. Thus NN is itself a "regimentation" of the Naiyāyikas' technical language.

7.3

One of the ideas which marked the passage from scholastic or medieval logic to the quantifier theory was the realisation that sentences should be seen as constructed in a series of stages, and not as constructed simultaneously from their component elements[16]. The two readings of "Someone loves everyone" are best distinguished if we do not regard the two expressions of generality and the relational expression as simultaneously synthesised; instead, we can see the sentence as built up from the relational expression "␣ loves ␣" in two steps. First, we form a predicate "someone loves ␣" or a predicate "␣ loves everyone", and then we fill in the remaining place. Dummett notes that in an ordinary language, there is an "ad hoc convention", that

> the order of construction corresponds to the inverse order of occurrence of the signs of generality in the sentence [7, p. 12].

This convention works because every sentence has both an active and a passive form, in which the order of the signs of generality are reversed. Thus, the active form "someone loves everyone" and the passive form "Everyone is loved by someone" are most naturally heard as expressing different readings of the sentence (although strictly each is ambiguous).

The Navya-Nyāya technical language seems, as I have tried to show, to encode the insight that sentences are constructed in stages. It is true that their language formalised the "ad hoc convention", and so lacked the elegance or clarity of a quantifier-variable system. Moreover, when a sentence is such that, in the quantifier system, several argument-places are filled by the same bound variable, the Nyāya language resorts, as does ordinary language, to the use of pronouns in order to generate an equivalent sentence

[16] Cf. [7, pp. 10*sqq*] and [10].

in which the sign of generality occurs only in a single place[17]. On the other hand, Dummett's criticism of natural languages, that they

> work by means of principles which are buried deep beneath the surface, and are complex and to a large extent arbitrary, [7, p. 20]

seems less applicable to the Nyāya language than to ordinary Sanskrit, for the principles to which it appeals are generally systematic, explicit, and, most importantly, unambiguous.

References

[1] Dinesh C. Bhattacharya. *History of Navya-Nyāya in Mithilā.* Mithilā Institute of Post-Graduate Studies and Research in Sanskrit Learning, Darbhanga, 1958.

[2] Kamaleswar Bhattacharyya. On the Language of Navya-Nyāya: An Experiment with Precision through Natural Language. *Journal of Indian Philosophy*, 34(1–2):5–13, 2006.

[3] Sibajiban Bhattacharyya. *Doubt, Belief and Knowledge.* Indian Council of Philosophical Research, New Delhi, 1987.

[4] Sibajiban Bhattacharyya. *Language, Testimony and Meaning.* Indian Council of Philosophical Research, New Delhi, 1998.

[5] Innocentius M. Bocheński. *A History of Indian Logic.* Chelsea Publishing Co., New York, 1961. Translated by Ivo Thomas.

[6] Mahesa Chandra Nyayaratna. *Brief Notes on the Modern Nyāya System of Philosophy and its Technical Terms.* Hare Press, Calcutta, 1891.

[7] Michael Dummett. *Frege. The Philosophy of Language.* Duckworth Press, London, 1981. 2nd Edition.

[8] Jonardon Ganeri. *Semantic Powers: Meaning and the Means of Knowing in Classical Indian Philosophy.* Clarendon Press, Oxford, 1999.

[9] Jonardon Ganeri. Towards a formal regimentation of the Navya-Nyāya technical language II. In THIS VOLUME, pages 125–141, 2008.

[10] Peter T. Geach. *Reference and Generality: An Examination of Some Medieval and Modern Theories.* Cornell University Press, Ithaca, 3 edition, 1980.

[17] Cf. [7, p. 13] and [15, p. 23].

[11] Cornelis Goekoop. *The Logic of Invariable Concomitance in the Tattvacintāmaṅi: Gangesa's Anumitinirupana and Vyaptivada with Introduction, Translation and Commentary.* D. Reidel, Dordrecht, 1967.

[12] Dinesh C. Guha. *Navya-Nyāya System of Logic: Some Basic Theories and Techniques.* Bharatiya Vidya Prakasan, Varanasi, 1968.

[13] Eberhard Guhe. George Bealer's Property Theories and their Relevance to the Study of Navya-Nyāya Logic. In THIS VOLUME, pages 143–157, 2008.

[14] Daniel H. H. Ingalls. *Materials for the Study of Navya-Nyāya Logic*, volume 40 of *Harvard Oriental Series.* Harvard University Press, Cambridge Massachusetts, 1951.

[15] Bimal Krishna Matilal. *The Navya-Nyāya Doctrine of Negation: The Semantics and Ontology of Negative Statements in Navya-Nyāya Philosophy*, volume 46 of *Harvard Oriental Series.* Harvard University Press, Cambridge, Massachusetts, 1968.

[16] Bimal Krishna Matilal. *Epistemology, Logic and Grammar in Indian Philosophical Analysis.* Mouton, The Hague, 1st edition, 1971.

[17] Bimal Krishna Matilal. *Logic, Language and Reality: Indian Philosophy and Contemporary Issues.* Motilal Banarsidass, Delhi, 1985.

[18] Karl H. Potter and Sibajiban Bhattacarya, editors. *Encyclopedia of Indian Philosophy. Volume 6: Indian Philososophical Analysis: Nyāya-Vaiśeṣika from Gaṅgeśa to Raghunātha Śiromaṇi.* Motilal BanarsiDass, Delhi, 1993.

[19] Willard V.O. Quine. *Word and Object.* MIT Press, Cambridge, 1960.

[20] Mark Sainsbury. *Russell.* Routledge and Kegan Paul, London, 1979.

[21] Saileswar Sen. *A Study on Mathurānātha's Tattvacintāmaṅi-rahasya.* G. van der Hoogt, Wageningen, 1924.

[22] Martin Stokhof. Hand or Hammer? On Formal and Natural Languages in Semantics. *Journal of Indian Philosophy*, 35(5–6):597–626, 2007.

[23] Toshihiro Wada. *The Analytical Method of Navya-Nyāya.* Egbert Forsten, Groningen, 2007.

[24] Ludwig Wittgenstein. *Philosophical Investigations.* Basil Blackwell, Oxford, 1958.